Further Pure 1 - January 2004

(1) $y_{n+1} = y_n + h f(x_n)$

$x_1 = 0$

$y_1 = 1$

$h = 0.2$

$y_2 = 1 + 0.2 \times \sqrt{1 + 0^2}$

$= 1.2$

$x_2 = 0.2$

$y_2 = 1.2$

$h = 0.2$

$y_3 = 1.2 + 0.2 \times \sqrt{1 + 0.2^2}$

$= 1.40396$ (5dp)

(2) a) $2 - 3i$ (conjugate)

b) $x^2 - \boxed{\text{SUM}}\, x + \boxed{\text{PRODUCT}} = 0$

$\boxed{\text{SUM}}$ $2 + 3i + 2 - 3i = 4 \Rightarrow b = -4$

$\boxed{\text{PRODUCT}}$ $(2+3i)(2-3i) = 4 + 9 = 13 \Rightarrow c = 13$

(3) tan: $\theta = n\pi + \alpha$

Key angle $(\alpha) = \tan^{-1}(\sqrt{3}) = \pi/3$

$\Rightarrow \pi/2 - 3x = n\pi + \pi/3$

$-3x = n\pi - \pi/6$

$\Rightarrow x = -n/3\,\pi + \pi/18$

which is same as $x = n/3 \cdot \pi + \pi/18$

(4) a) $\Sigma r = 1/2\, n(n+1)$

$\Sigma r^2 = 1/6\, n(n+1)(2n+1)$

$\Rightarrow 3\Sigma r^2 - 3\Sigma r + \Sigma 1$

$= 1/2\, n(n+1)(2n+1) - 3/2\, n(n+1) + n$

$= 1/2\, n\left[(n+1)(2n+1) - 3(n+1) + 2\right]$

$= 1/2\, n\left[2n^2 + 3n + 1 - 3n - 3 + 2\right]$

$= 1/2\, n\left[2n^2\right] = n^3$

b) $\sum_{n+1}^{2n} = \sum_{1}^{2n} - \sum_{1}^{n}$

$= (2n)^3 - (n)^3 = 7n^3$

3 a) i) $A + B = \begin{bmatrix} k & k \\ k & -k \end{bmatrix} + \begin{bmatrix} -k & k \\ k & k \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix}$

ii) A^2

$$\begin{array}{c|c} & \begin{bmatrix} k & k \\ k & -k \end{bmatrix} \\ \hline \begin{bmatrix} k & k \\ k & -k \end{bmatrix} & \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix} \end{array}$$

b) $(A+B)^2 =$

$$\begin{array}{c|c} & \begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix} & \begin{bmatrix} 4k^2 & 0 \\ 0 & 4k^2 \end{bmatrix} \end{array}$$

$B^2 =$

$$\begin{array}{c|c} & \begin{bmatrix} -k & k \\ k & k \end{bmatrix} \\ \hline \begin{bmatrix} -k & k \\ k & k \end{bmatrix} & \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix} \end{array}$$

$A^2 = \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix}$

$\therefore A^2 + B^2 = \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix} + \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix} = \begin{bmatrix} 4k^2 & 0 \\ 0 & 4k^2 \end{bmatrix}$

$= (A+B)^2$

c) i) $A^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ = Enlargement, scale factor 2, centre (0,0)

ii) $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ Enlargement SF must $= \sqrt{2}$

$\rightarrow \quad \sqrt{2}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

Reflection

$\cos(2\theta) = 1/\sqrt{2}$

$\rightarrow 2\theta = 45°$

$\rightarrow \theta = 22.5°$

$\therefore$ Reflection in line $y = \tan(22.5)x$

(6) a) i) Denominator = 0 when $x = 0$ & $x = 2$

$\rightarrow x = 0, \ x = 2$

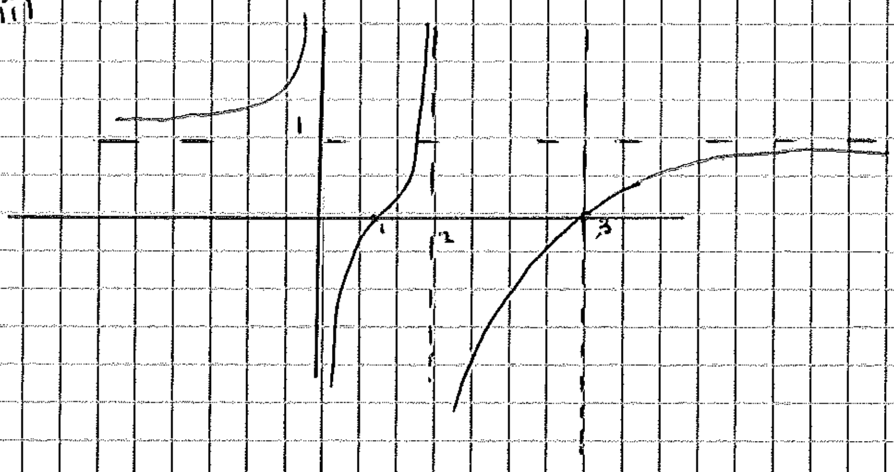
As $x \rightarrow \infty$, $y \rightarrow 1/1 \quad \rightarrow y = 1$

ii) $y = 0 \rightarrow 0 = \dfrac{(x-1)(x-3)}{x(x-2)}$

$\rightarrow 0 = (x-1)(x-3)$

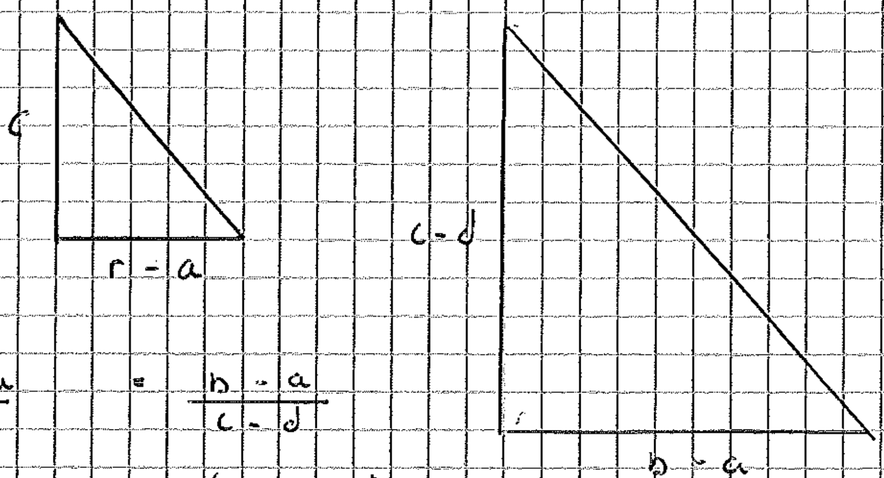
$\rightarrow x = 1$ & $x = 3$ or $(1,0)$ & $(3,0)$

iii)



b) curve below x-axis when: $0 < x < 1$ AND $2 < x < 3$

(7) c)



$\dfrac{r-a}{c} = \dfrac{b-a}{c-d}$

$r - a = c\left(\dfrac{b-a}{c-d}\right)$

$\rightarrow r = a + c\left(\dfrac{b-a}{c-d}\right)$

b) i) $a = 2 \rightarrow c = 20(2) - (2)^4 = 24$

$b = 3 \rightarrow d = 20(3) - 3^4 = -21$

$\rightarrow r = 24\left(\frac{3-2}{24-(-21)}\right) + 2 = 38/15$

ii) at β, $f(x) = 0$

$\rightarrow 20x - x^4 = 0$

$\rightarrow x^4 - 20x = 0$

$x(x^3 - 20) = 0$

$x = 0$
This is not β

$x^3 - 20 = 0$

$\rightarrow x = \sqrt[3]{20} = \beta$

$\therefore \beta - r = \sqrt[3]{20} - 38/15 = 0.181...$

(8) a) $\int_1^\infty x^{-3/4}\,dx = \int_1^n x^{-3/4}\,dx$

$= \left[4x^{1/4}\right]_1^n = 4\sqrt[4]{n} - 4\sqrt[4]{1}$

As $n \rightarrow \infty$, $4\sqrt[4]{n}$ does not converge to a limit, so $\int$ has no value

b) $\int_1^\infty x^{-5/4}\,dx = \int_1^n x^{-5/4} = \left[-4x^{-1/4}\right]_1^n$

$= \frac{-4}{\sqrt[4]{n}} - \left(\frac{-4}{1}\right)$

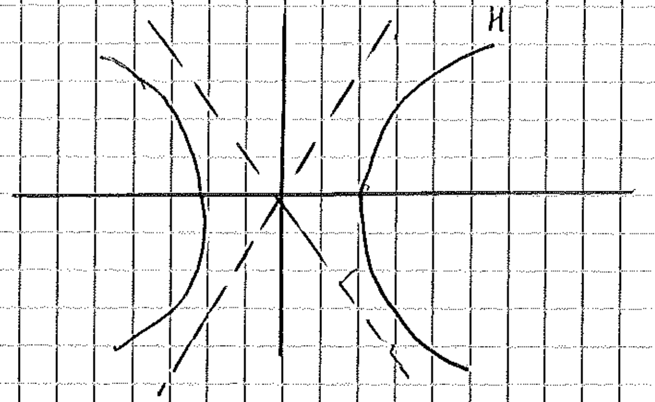
As $n \rightarrow \infty$, $\frac{-4}{\sqrt[4]{n}} \rightarrow 0$, and $\therefore \int \rightarrow 4$

c) First component still has no value, so subtracting 4 makes no difference. = NO VALUE.

(9) a) From formula book, asymptotes $\frac{x}{1} = \pm \frac{y}{\sqrt{2}}$

$\rightarrow y = \sqrt{2}\,x$ and $y = -\sqrt{2}\,x$

b)



c) i) $y = x + c$ $\quad \Rightarrow \quad x^2 - \dfrac{(x+c)^2}{2} = 1$

$$\Rightarrow \quad 2x^2 - (x+c)^2 = 2$$

$$2x^2 - x^2 - 2xc - c^2 = 2$$

$$x^2 - 2cx - (c^2+2) = 0$$

ii) Discriminant: $b^2 - 4ac$

$$= (-2c)^2 - 4 \times 1 \times -(c^2+2)$$

$$= 4c^2 + 4c^2 + 8$$

$$= 8c^2 + 8$$

whatever the value of c, this is always +ve, so 2 distinct solutions

iii) Solve quadratic:

$$x = \frac{2c \pm \sqrt{8c^2+8}}{2}$$

$$x = \frac{2c \pm \sqrt{4}\sqrt{2c^2+2}}{2}$$

$$x = c \pm \sqrt{2c^2+2}$$

$y = x + c$

$$\Rightarrow y = 2c \pm \sqrt{2c^2+2}$$